Axial and tensor charge of the nucleon in the Dirac orbital model

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Abstract

Using the expansion of the baryon wave function in a series of products of single quark bispinors (Dirac orbitals), the nonsinglet axial and tensor charge of the nucleon are calculated. The leading term yields $G_A/G_V = 1.27$ and in good agreement with experiment. Calculation is essentially parameter-free and depends on the string tension σ and α_s, fixed at standard values. The importance of lower Dirac bispinor component, yielding 18% to the wave function normalization is stressed.

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Axial and tensor charges of nucleons are important to characterize the basic structure of the nucleon as composed of strongly coupled quarks [1]-[3]. It is known that nonrelativistic quark models predict $\frac{G_A}{G_V} = \frac{5}{3}$ in strong disagreement with experimental value 1.27, while for massless relativistic quarks, e.g. in the MIT bag model, one obtains much smaller values $\frac{G_A}{G_V} = 1.09$. Thus the calculation of $\frac{G_A}{G_V}$ (and tensor charge δq) gives a clue to the relativistic dynamics of quarks in the nucleon.

Moreover, in a recent paper [4] it was shown that the knowledge of the ratio of $\frac{G_A}{G_V}$ for baryon decays is important for the accurate determination of the CKM matrix element V_{us} These considerations justify the systematic analysis of baryon decays in the framework of Dirac Orbital Expansion (DOE) the first part of which is reported below.

The contribution of vector and axial hadronic currents, $V_\mu = i\bar{\psi}_u \gamma_\mu \psi_d$, $A_\mu = i\bar{u}\gamma_\mu\gamma_5\psi_d$, to the neutron β-decay is characterized by the ratio

$$\frac{G_A}{G_V} = \frac{\langle p_\uparrow \mid A_z \mid n_\uparrow \rangle}{\langle p_\uparrow \mid V_0 \mid n_\uparrow \rangle}, \tag{1}$$

where $p_\lambda, n_\lambda, \lambda = \pm\frac{1}{2}$ are proton and neutron wave functions with spin projection λ [1]-[3]. In a similar way the tensor charge is expressed through the proton matrix element of the tensor current $T_{\mu\nu}\bar{\psi} i\sigma_{\mu\nu}\gamma_5\psi$ [5].

To construct the baryon wave function, one starts with the Hamiltonian [6] obtained in the instantaneous approximation from the general Bethe–Salpeter equation:

$$\hat{H}\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = E\Psi, \quad \hat{H} = \sum_{i=1}^{3} \hat{H}_i + \Delta H \tag{2}$$

with

$$\hat{H}_i = \mathbf{p}_{(i)}\boldsymbol{\alpha}_{(i)} + \beta_{(i)}(m_i + M(\mathbf{r}_i)) \tag{3}$$

where $M(\mathbf{r}_i)$ in the limit of vanishing gluon correlation length is

$$M = \sigma|\mathbf{r}_i|e^{i\gamma_5\hat{\phi}(\mathbf{r}_i)}$$

and $\mathbf{r}_i = \mathbf{x}_i - \mathbf{x}_0$, $\mathbf{x}_0$ is the string-junction coordinate and $\hat{\phi}(\mathbf{r}_i)$ is the Nambu–Goldstone octet. Here ΔH contains perturbative gluon exchanges. We expand the baryon wave function in a series of products of quark eigenfunctions $\psi_n^{(i)} = \binom{v^{(i)}}{w^{(i)}}$, namely [7, 8]

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sum_{\{n_i\}} \prod_{i=1}^{3} \psi_{n_i}^{(i)}(\mathbf{r}_i) C_{n_1 n_2 n_3} \tag{4}$$

In what follows we shall consider the leading valence approximation for the nucleon keeping only the first term in (4), $\Psi \to \Psi_0$, which contains the ground state S–wave Dirac orbitals $|u_\lambda\rangle$ and $|d_\lambda\rangle$ for u and d quarks with spins up and down. One has

$$|p\uparrow\rangle = \sqrt{\frac{1}{18}}[-2(|u\uparrow u\uparrow d\downarrow\rangle + perm.) + (|u\uparrow u\downarrow d\uparrow\rangle + perm.)] \tag{5}$$

$$|n\uparrow\rangle = \sqrt{\frac{1}{18}}[-2(|d\uparrow d\uparrow u\downarrow\rangle + perm.) + (|d\uparrow d\downarrow u\uparrow\rangle + perm.)] \tag{6}$$

Table 1: G_A/G_V and η for various theoretical prescriptions in comparison with experimental data

	Exp.	NRQM	$\zeta=0$	$\zeta=0.3$
G_A/G_V	1.27	1.67	1.36	1.27
η	–	0	0.14	0.18

The expressions (5) and (6) have the same form as in the standard $SU(4)$ or $SU(6)$ model [3] except for the bispinor contents of $|u_\lambda\rangle$ and $|d_\lambda\rangle$.

Insertion of (5), (6) into (1) yields[1]

$$\frac{G_A}{G_V} = +\frac{5}{3}\Big\langle \chi_\uparrow \Big| \mathbf{\Sigma}_3 \Big| \chi_\uparrow \Big\rangle, \quad \mathbf{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} \tag{7}$$

where $\chi_\uparrow$ is

$$\chi_\uparrow(r,\theta,\phi) = \frac{1}{r}\begin{pmatrix} G(r)\Omega_{\frac{1}{2},0,\frac{1}{2}}(\theta,\phi) \\ iF(r)\Omega_{\frac{1}{2},1,\frac{1}{2}}(\theta,\phi) \end{pmatrix}; \quad \int_0^\infty \left[G^2(r) + F^2(r)\right] dr = 1 \tag{8}$$

To take into account perturbative gluon exchange we represent ΔH effectively as one-particle operators,

$$\Delta H = \sum_{i=1}^{3}\left(-\frac{\zeta}{r_i}\right)$$

and the equations for $G(r)$, $F(r)$ acquire the form [10]

$$\begin{aligned} G' - \frac{1}{r}G - \left(E + m + \sigma r + \frac{\zeta}{r}\right) F = 0, \\ F' + \frac{1}{r}F + \left(E - m - \sigma r + \frac{\zeta}{r}\right) G = 0 \end{aligned} \tag{9}$$

Finally G_A can be written as

$$G_A = +\frac{5}{3}\left(1 - \frac{4}{3}\eta\right), \quad \eta = \int_0^\infty F^2(r)dr \tag{10}$$

We have computed the values of η and for $m=0$ and two different values of ζ: $\zeta = 0$ and $\zeta = 0.3$. The results are given in Table 1

Note that for $m = 0$ G_A does not depend on the string tension σ on dimensional grounds. One can see that in the Table 1 that the resulting G_A is in the correct ballpark for $\zeta \in [0, 0.3]$. The value $\zeta = 0.3$ corresponds to the reasonable effective value of α_s in the qq potential, namely from $\left\langle \sum \frac{2}{3}\frac{\alpha_s}{r_{ij}} \right\rangle = \left\langle \sum \frac{\zeta}{r_i} \right\rangle$, and $\langle r_{ij}\rangle \approx \sqrt{3}\langle r_i\rangle$, one has

$$(\alpha_s)_{\text{eff}} = \frac{3\sqrt{3}}{4}\zeta \approx 0.39$$

[1] The sign of G_A corresponds to [1, 9]

and this value of ζ was checked in the actual calculation of the nucleon mass [11]. It is rewarding that the resulting $G_A = 1.27$ is in close agreement with experiment.

Concerning the tensor charge δq, it can be easily calculated the definition of δq, and in the same way as in Eq. (7) one arrives at he expression

$$\delta q = \left\langle \chi_\uparrow \,\middle|\, \beta \mathbf{\Sigma}_3 \,\middle|\, \chi_\uparrow \right\rangle \tag{11}$$

As a result one has for $\zeta = 0.3$:

$$\delta q = \delta u - \delta d = \frac{5}{3}\left(1 - \frac{2}{3}\eta\right) \approx 1.47 \tag{12}$$

Note that in the nonrelativistic limit $\delta q = G_A$.

One can compare these results with the lattice data [12], where both G_A and δq are close to each other and are in the interval $1.12 \leq G_A, \delta q \leq 1.18$ [12] for $m_\pi > 0.5$ GeV.

Calculations of δq in other methods give results ranging from 1.07 to 1.45, see [5] for refs. and discussion. Note that the anomalous dimension of the tensor charge is small and calculations here and in [5] refer to the scale $\mu^2 = M_N^2$.

There are two possible unaccounted effects which can influence our results. First, the contribution of other terms in (4) – excited Dirac orbitals. The corresponding multichannel calculations done in [8] for magnetic moments, result in decreasing of the modulus of magnetic moments of proton and neutron by some $10 - 15\%$ when one accounts for 4 Dirac orbitals for each quarks, and one can expect the same type of corrections for G_A. Second, the contribution of chiral degrees of freedom, i.e. of the π, η, K exchanges. Again, for nucleon magnetic moments these corrections are typically of the order of 10% [8], and we expect this to be an upper limit for G_A, since magnetic moments are much more sensitive to the contribution of the lowest Dirac components, than G_A and δq, where these contributions enter quadratically and not linearly.

These corrections are not taken into account above, which is planned for a subsequent work, where also hyperon semileptonic decays are considered [13].

One should note, that relativistic approach to the baryon wave function, based on the light-cone formalism [14] was shown to improve the qualitative agreement of G_A with experiment, however quantitatively still far from experiment.

Summarizing, we have calculated nonsinglet axial and tensor charges in the simple relativistic model of the nucleon, where the wave function is a product of three Dirac orbitals of quarks. The resulting value of G_A is in excellent agreement with experiment for the choice of the only parameter $(\alpha_s)_{\text{eff}} = 0.39$, yielding the reasonable value of the nucleon mass. Note that quarks in the baryon prove rather relativistic, so the contribution of the lower quark bispinor component to G_A is not negligibly small: $\eta \approx 0.18$.

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